Work

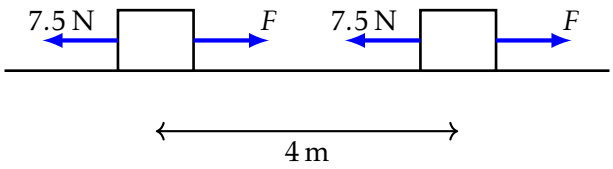
Fact —

$$\text{Work Done} = \underbrace{\text{Force}}_{\text{component of force in the direction of travel}} \times \text{Distance}$$

What are the units of work?

Example

A block of wood is pulled a distance of 4 m across a horizontal surface against resistances totalling 7.5 N. If the block moves at a constant velocity, find the work done against the resistance.

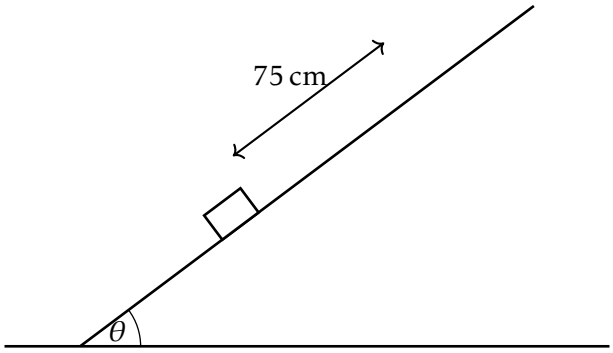


Example

A rough surface is inclined at $\tan^{-1}\frac{7}{24}$ to the horizontal. A body of mass 5 kg lies on the surface and is pulled at a uniform speed a distance of 75 cm up the surface by a force acting along a line of greatest slope. The coefficient of friction between the body and the surface is $\frac{5}{12}$. Find:

(a) the work done against gravity

(b) the work done against friction.



Energy

What is energy?

Kinetic Energy

Example

Suppose a constant force F acts on a particle of mass m, over a distance s, accelerating the particle to a speed v. What is the work done on the body?

Gravitational Potential Energy

Fact — The work done moving a body of mass m through a height h against gravity g is mgh. We can pick a height to be our 0 G.P.E. level and then measure gravitational potential energy from here.

Conservation of Energy

Why do we care about these different energies? Because of an extremely important result:

Fact (Principle of Conservation of Mechanical Energy) — Assuming gravity is the only external force which does work on the body, then the total energy possessed by the body is constant, ie:

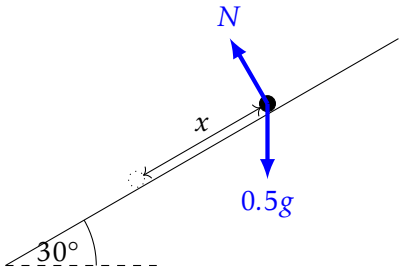
$$\begin{aligned}\text{total energy} &= \underbrace{\text{kinetic energy}}_{K.E.} + \underbrace{\text{potential energy}}_{P.E.} \\ &= \text{constant}\end{aligned}$$

alternatively:

$$\text{Initial energy} = \text{Final Energy}$$

Example

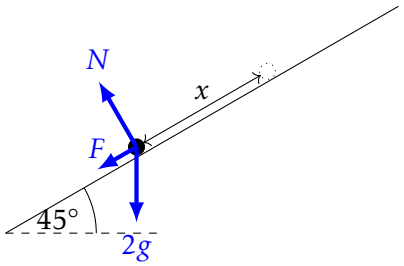
A smooth plane is inclined at 30° to the horizontal. A particle of mass 0.5 kg slides down a line of greatest slope of the plane. The particle starts from rest at point A and passes point B with a speed $6\,\text{m}\,\text{s}^{-1}$. Find the distance AB.



Fact (Work-Energy principle) — The change in the total energy of a particle is equal to the work done on the particle.

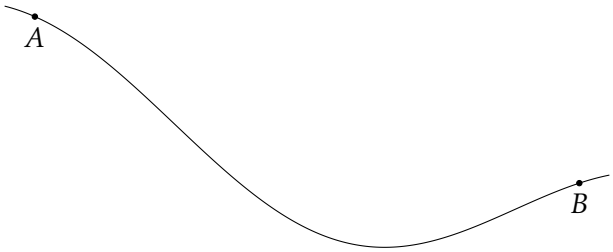
Example

A particle of mass 2 kg is projected with speed $8\,\mathrm{m\,s^{-1}}$ up a line of greatest slope of a rough plane inclined at 45° to the horizontal. The coefficient of friction between the particle and the plane is 0.4. Calculate the distance the particle travels up the plane before coming to instantaneous rest.



Example

A skier passes a point A on a ski-run moving downhill at $6\,\text{ms}^{-1}$. After descending 50 m vertically the run begins to ascend. When the skier has ascended 25 m to a point B her speed is $4\,\text{ms}^{-1}$. The skier and her skis have a combined mass of 55 kg. The total distance she travels from A to B is 1400 m. The non-gravitational resistance to motion are constant and have a total magnitude of 12 N. Calculate the work done by the skier.

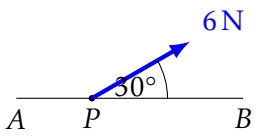


Work done by forces at angles

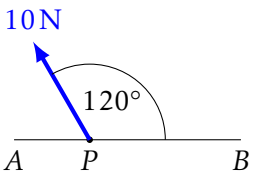
Example

The diagrams below show a constant force acting on a particle P. The force continues to act as the particle is made to move along a straight line path from A to B, a distance of 3 m. Find the work done by the force in each case.

(a)



(b)

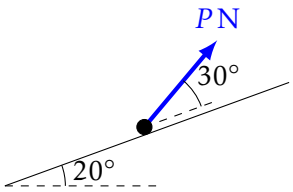


Example (OCR M2 Jan 2007 Q4)

A skier of mass 80 kg is pulled up a slope which makes an angle of 20° with the horizontal. THe skier is to a constant frictional force of magnitude 70 N. The speed of the skier increases from 2 ms^{-1} at the point A to 5 ms^{-1} at the point B, and the distance AB is 25 m.

(i) By modelling the skier as a small object, calculate the work done by the pulling force as the skier moves from A to B.

(ii) It is given that the pulling force has constant magnitude PN, and that it acts at a constant angle of 30° above the slope (see diagram). Calculate P.



Power

Fact —

$$\begin{aligned}\text{Power} &= \frac{\text{Work done}}{\text{time}} \\ &= \text{the 'rate' of doing work} \\ &= \frac{\mathrm{d}}{\mathrm{d}t}(\text{Work done})\end{aligned}$$

If the force being applied is constant, then

What are the units of power?

Example

A lift, of total mass 1200 kg, rises a distance of 60 m in 20 s. What is the power output of the motor?

Example

A van of mass 1250 kg is travelling along a horizontal road. The van's engine is working at 24 kW. The constant resistance to motion has a magnitude 600 N. Calculate

(a) the acceleration of the van when it is travelling at 6 ms^{-1},

(b) the maximum speed of the van

Fact —

$$\text{Power} = \text{Force} \times \text{velocity}$$